SUBLIMATION OF A THIN PLATE IN A GAS STREAM

V. G. Leitsina and N. V. Pavlyukevich

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The laminar boundary layer on a sublimating thin plate is examined.

When a gas stream flows over a flat plate and the partial pressures of the vapors of the plate material are less than the pressure at the triple point of the phase diagram, sublimation or condensation of the vapors occurs (depending on the parameters of the oncoming stream and the heat transfer conditions at the surface).

We shall examine sublimation of a thin plate washed by a stream of heated gas. The lower surface of the plate is maintained at constant temperature.

We choose the x and y axes along and normal to the original plate surface, respectively. Then the system of equations of the laminar boundary layer for the binary gas-vapor mixture has the following form:

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{v}) = 0, \tag{1}$$

$$\rho \frac{du}{dt} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right), \tag{2}$$

$$\rho \frac{dc}{dt} = \frac{\partial}{\partial y}\left(\rho D_{12} \frac{\partial c}{\partial y}\right), \tag{3}$$

$$\rho \frac{dH}{dt} = \frac{\partial}{\partial y}\left\{\frac{\mu}{\Pr}\left[\frac{\partial H}{\partial y} + (\Pr - 1)\frac{\partial}{\partial y}\left(\frac{u^2}{2}\right) + \left(\frac{1}{\mathrm{Le}} - 1\right)(h^{(1)} - h^{(2)})\frac{\partial c}{\partial y}\right]\right\}. \tag{4}$$

The heat conduction equation in the plate must be solved along with the system (1)–(4). Since the plate is considered sufficiently thin ($x \gg l$), we shall seek a temperature distribution in it of the form

$$T_1(x, y, t) = T_0 - q(x, t)(y - Dt), \tag{5}$$

where

$$\left.\frac{\partial T_1}{\partial y}\right|_{y=Dt} = -q.$$

We write the boundary conditions of the problem as [1] in the flow within the boundary layer

$$u = u_\infty, \quad T = T_\infty, \quad c = c_\infty, \tag{6}$$

at the sublimation surface

$$\rho(D - v)(1 - c_0) = \rho D_{12} \frac{\partial c}{\partial y}, \tag{7}$$

$$\rho(D - v) = \rho_1 D, \quad u = 0, \quad T = T_1 = T_0, \tag{8}$$

$$\rho(D - v) L(T_0) = \lambda_1 \frac{\partial T_1}{\partial y} - \lambda \frac{\partial T}{\partial y}, \tag{9}$$

$$c_0 = \left[1 + \left(\frac{p_\infty}{p} - 1\right)\frac{M_2}{M_1}\right]^{-1}, \tag{10}$$

at the lower surface of the plate ($y = -l$)

$$T_1 = T_1^*. \tag{11}$$

Here

$$h = c_p T, \quad c_p = c c_p^{(1)} + (1 - c) c_p^{(2)}, \quad H = h + u^2/2.$$

We seek a solution of the equation of motion (2) in the form [1]

$$\rho u = \varphi'_\eta(\eta, t),$$

$$\rho v = \frac{1}{2}\left(\frac{\nu_\infty}{u_\infty x}\right)^{1/2}\left[\eta\varphi'_\eta - \varphi(\eta, t) + \frac{\rho}{\rho_1}\varphi(0, t)\right],$$

$$\rho_1 D = \frac{1}{2}\left(\frac{\nu_\infty u_\infty}{x}\right)^{1/2}\varphi(0, t),$$

where the velocities u and v and the densities ρ and ρ_1 are measured, respectively, at the values u_∞ and ρ_∞.

As was shown in [1], for sufficiently small times (on the order of several tens of seconds), the rate of displacement D of the sublimation surface may be considered to be dependent only on the coordinate x, i.e., a steady sublimation regime is established at each point on the surface

$$D = \frac{1}{2}\frac{\rho_\infty}{\rho_1}(\nu_\infty u_\infty)^{1/2}\frac{\varphi(0)}{\sqrt{x}}$$

or

$$D = \frac{\alpha}{\sqrt{x}}\varphi(0), \tag{12}$$

where

$$\alpha = \frac{1}{2}\frac{\rho_\infty}{\rho_1}(\nu_\infty u_\infty)^{1/2}.$$

Then, for the above times, the concentration of the vapors of the subliming material at the surface c_0, and the surface temperature T_0 (we consider the vaporization to be in equilibrium [2]) will also be independent of time. Therefore, the function q in (5) will also depend only on x, since, from condition (11), taking into account the inequality $Dt \ll l$, we obtain

$$q = (T_1^* - T_0)/l. \tag{13}$$

Because of the assumption of steady conditions of sublimation, equation (2) reduces to the Blasius equation [1]

$$\frac{d}{d\eta}\left(\mu' \frac{du}{d\eta}\right) + \frac{\varphi}{2} \frac{du}{d\eta} = 0;\ \mu' = \frac{\mu}{\mu_\infty}. \qquad (14)$$

The solutions of this equation, allowing for the first of the boundary conditions (6), has been obtained in [3] ($\rho\mu' = 1$)

$$\omega(u) = u_0^{-3/2} \omega_\gamma(uu_0),\ \eta = \int_0^u \frac{\mu}{\omega(u)} du.$$

Here

$$\omega = \mu' \frac{du}{d\eta},\quad \eta = \sqrt{\frac{u_\infty}{\nu_\infty x}}\,(y - Dt),$$

and the tables of functions $u_0(\gamma)$ and $\omega_\gamma(u)$ were given in [3].

We shall restrict attention to the case Pr = Le = 1. Then the solutions of the equations of diffusion (3) and energy (4), taking into account boundary conditions (6), have the following form:

$$c = c_0 + (c_\infty - c_0)\, u(\eta)/u_\infty,$$

$$h = h_0 + (H_\infty - h_0)\, u(\eta)/u_\infty - \frac{1}{2} u^2(\eta).$$

Thus, to determine the profiles of velocity, concentration, and enthalpy, it is necessary to find the quantities u_0, tg γ, c_0 and T_0.

It follows from (14) that

$$\varphi(0) = -2 \left.\frac{d\omega}{du}\right|_{u=0} = -2u_0^{-1/2}\,\mathrm{tg}\,\gamma.$$

Substitution of this expression in (12) gives

$$D = -2a\, u_0^{-1/2}\,\mathrm{tg}\,\gamma/\sqrt{x}. \qquad (15)$$

From (7) and (8) we obtain

$$u_0\,\mathrm{tg}\,\gamma = \frac{c_0 - c_\infty}{1 - c_0}. \qquad (16)$$

Condition (9) reduces to the following:

$$-\lambda_1 q/\rho_1 D + \{(1-c_0)(h_0^{(1)} - h_0^{(2)}) - (1-c_0)[H_\infty - c_0 h_0^{(1)} - (1-c_0) h_0^{(2)}]/(c_\infty - c_0)\} \times \left\{1 + \frac{\partial(c_p^{(1)} - c_p^{(2)})}{\partial T} c_0 T_0/c_p + \frac{\partial c_p^{(2)}}{\partial T} T_0/c_p\right\}^{-1} - L(T_0) = 0. \qquad (17)$$

At comparatively small pressures a saturated vapor may be considered to be a perfect gas. In this case, from the Clausius-Clapeyron equation, we have for the heat of sublimation the expression

$$L = -R \frac{d(\ln p)}{d(1/T)}. \qquad (18)$$

Let $c_\infty = 0$. Then it follows from (16) that

$$u_0\,\mathrm{tg}\,\gamma > 0.$$

Since the function u_0 is always positive, tg $\gamma > 0$. For positive tg γ, using tables [3], we may approximate to function u_0, for example, by the method of least squares

$$u_0 = 2.0792 \exp\{1.024\,\mathrm{tg}\,\gamma\}. \qquad (19)$$

Using (13), (15), (18) and (19), we finally obtain from (10), (16), and (17) a system of equations for determining c_0, T_0 and tgγ

$$c_0 = [1 + (p_\infty/p - 1) M_2/M_1]^{-1}, \qquad (20)$$

$$2.0792 \exp\{1.024\,\mathrm{tg}\,\gamma\} = c_0/(1 - c_0), \qquad (21)$$

$$\frac{0.721\lambda_1\sqrt{x}(T_1^* - T_0)\exp\{0.512\,\mathrm{tg}\,\gamma\}}{a\rho_1 l\,\mathrm{tg}\,\gamma} + \{(1-c_0)T_0(c_p^{(1)} - c_p^{(2)}) + (1-c_0)[H_\infty - c_0 T_0 c_p^{(1)} - (1-c_0)T_0 c_p^{(2)}]/c_0\} \times \left[1 + \frac{\partial(c_p^{(1)} - c_p^{(2)})}{\partial T} c_0 T_0/c_p + \frac{\partial c_p^{(2)}}{\partial T} T_0/c_p\right]^{-1} = -R\frac{d(\ln p)}{d(1/T)}. \qquad (22)$$

We shall examine as an example the equilibrium sublimation of a thin plate of naphthalene in a uniform stream of heated air. The pressure of the saturated vapors of naphthalene may be represented in the form

$$p = \exp\{a - b/(d + T_0)\}. \qquad (23)$$

The coefficients a, b, and d are determined by scaling the coefficients of the Antoine equation [4]. From (18) and (23) we determine the heat of sublimation to be

$$L = bR \Big/ \left(\frac{d}{T_0} + 1\right)^2. \qquad (24)$$

We take the specific heat of air $c_p^{(2)}$ to be independent of temperature; at the sublimation surface it is equal [5] to 1009 J/kg · degree. The specific heat of naphthalene vapor [6] is

$$c_p^{(1)} = A + BT + CT^{(2)}. \qquad (25)$$

The coefficients in (23) and (25) have the following values (all quantities are measured in the international system):

$$a = 18.268;\ b = 2253.419;\ d = -154.76;\ A = 102.903;$$
$$B = 3.574;\ C = 1.136\cdot 10^{-3}.$$

We shall assign the conditions at infinity in the stream as: $u_\infty = 400$ m/sec; $T_\infty = 900°$ K; $c_\infty = 0$; $p_\infty = 6650$ N/m^2. From these conditions we determine [5] $\rho_\infty = 0.0258$ kg/m^3; $\mu_\infty = 398 \cdot 10^7$ N · sec/m^2 and calculate $H_\infty = 1\,089\,856$ J/kg.

Let the plate thickness $l = 0.01$ m, and the temperature at the lower surface of the plate be $T_1^* = 290°$ K.

For naphthalene [7] $\rho_1 = 1168$ kg/m^3; $\lambda_1 = 376{,}812 \cdot 10^3$ W/m · deg. The molecular weights of air and naphthalene were assumed to be as follows [5]: $M_2 = 28.96$ kg/kmole, $M_1 = 128.164$ kg/kmole.

After substitution of (23)–(25) and numerical values, the system of Eqs. (20), (22) is solved as follows: we assign a value of the concentration c_0, we determine T_0 from (20), and $tg\gamma$ from (21); then equation (22), in which x appears as a parameter, allows us to determine the value of x corresponding to the given combination T_0, c_0, $tg\gamma$.

From (15), allowing for (19), and after substituting $tg\gamma$, we obtain the rate of displacement of the sublimation surface.

The results of the calculations are

x, m	c_0	T_0, °K	$-D\cdot 10^5$, m/sec
0.091	0.44	353.2	0.975
0.215	0.40	350.7	0.568
0.460	0.35	347.6	0.326
0.917	0.30	344.2	0.199

Formula (5) allows the temperature inside the plate to be calculated.

NOTATION

u and v—longitudinal and transverse velocity components; ρ and ρ_1—densities of gas-vapor mixture and plate material, respectively; λ and λ_1—thermal conductivities of mixture and plate material, respectively; μ—viscosity of mixture; c_p—specific heat at constant pressure; h—specific enthalpy of mixture; $Pr = \mu c_p/\lambda$—Prandtl number; $Le = \lambda/\rho c_p D_{12}$—Lewis number; c—vapor mass concentration; D_{12}—binary diffusion coefficient; φ—stream function; D—rate of displacement of sublimation surface; M_1 and M_2—molecular weight of plate material and gas, respectively; T_1—variable temperature within plate; l—plate thickness; R—universal gas constant; L—heat of sublimation of plate material. Subscripts: ∞—flow quantities inside boundary layer, 0—parameters on sublimation surface. Superscripts: 1 and 2—parameters of vapor and gas, respectively.

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Institute of Heat and Mass Transfer, AS BSSR, Minsk